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## The psychology of indicative conditionals and conditional bets

**Abstract.** There is a new Bayesian, or probabilistic, paradigm in the psychology of reasoning, with new psychological accounts of the indicative conditional of natural language. In psychological experiments in this new paradigm, people judge that the probability of the indicative conditional,  $P(\text{if } A \text{ then } C)$ , is the conditional probability of  $C$  given  $A$ ,  $P(C | A)$ . In other experiments, participants respond with what has been called the ‘defective’ truth table: they judge that *if  $A$  then  $C$*  is true when  $A$  holds and  $C$  holds, is false when  $A$  holds and  $C$  does not, and is neither true nor false when  $A$  does not hold. We argue that these responses are not ‘defective’ in any negative sense, as many psychologists have implied. We point out that a number of normative researchers, including de Finetti, have proposed such a table for various coherent interpretations of the third value. We review the relevant general tables in the normative literature, in which there is a third value for  $A$  and  $C$  and the logically compound forms of the natural language conditional, negation, conjunction, disjunction, and the material conditional. We describe the results of an experiment on which of these tables best describes ordinary people’s judgements when the third value is interpreted as indicating uncertainty.

**Keywords:** Bayesian account of reasoning; probability conditional; uncertainty and three-valued tables; de Finetti tables

### Introduction and overview

Researchers working in the field of the psychology of reasoning have generally selected some theoretical model to establish a referential norm of ‘rational inference’. Many psychological studies consist of comparing participants’ responses to the results prescribed by such a normative model. Psychologists have traditionally assumed that there are different psychological processes corresponding to the subject divisions of the field: judgement and decision-making, probability judgement / inductive reasoning, and deductive reasoning. The three major normative models used are (i) the *Subjective Expected Utility Theory* in decision theory studies, (ii) the *Bayesian model* in the context of probability judgement and induction (iii) extensional bivalent *Propositional Logic* for deductive reasoning. However the choice of a specific normative model for a given field has deep epistemological implications (for probability judgement see [2, 5, 6]). More drastically, theorists have

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increasingly objected to the very segmentation of inferences in the traditional approach. Indeed, there is a *new Bayesian paradigm* that seeks to integrate the psychology of reasoning. It holds that people, in everyday and scientific contexts, tend to reason under uncertainty even when carrying out a deductive task. In this new paradigm, *rationality* is defined in terms of Bayesian probability theory rather than with reference to extensional logic ([54, 55, 58]). But a great deal of psychological research will be necessary to identify the logic system(s) underlying everyday reasoning in natural language and to determine its compatibility with Bayesian theory. The present paper aims to sketch a method to identify the kind of logic that underlies lay peoples' reasoning; it adopts the standpoint of the new paradigm and introduces uncertainty as a third value<sup>1</sup> in addition to truth and falsity.

Two experimental findings have given considerable impetus to the new Bayesian paradigm in the psychology of reasoning and its aim of explaining reasoning under uncertainty ([3, 33, 54, 55, 58, 60]).

The first finding is the confirmation of the *conditional probability hypothesis* that people judge the probability of the natural language indicative conditional, *if A then C*, to be the conditional probability of *C* given *A*,  $P(C | A)$  and not the probability of the material conditional ( $P(A \supset C)$ ) equivalent to *not-(A & not-C)*, as commonly assumed in the old paradigm. This relationship,  $P(\text{if } A \text{ then } C) = P(C | A)$ , has such far reaching implications that it is sometimes called *the Equation* in both philosophy ([31]) and psychology ([54, 55]). It has been strongly supported in a wide range of experiments ([27, 34, 35, 39, 56, 59, 61]).

The second finding is what has long been called the *defective truth table* in psychology (see [36]). Participants are given *truth table tasks* and asked to make a judgement about a natural language indicative conditional, *if A then C*, when given rows of the table. The main result of these experiments is that people do not give the material conditional truth table. They judge that *if A then C* is true when *A & C* holds and false when *A & not-C* holds, but that *not-A & C* and *not-A & not-C* are irrelevant to the truth value of *if A then C*, and that *if A then C* is neither true nor false when either of these *not-A* rows hold (see [61]).

---

<sup>1</sup>We will restrict ourselves to three values in this paper and so not divide uncertainty into degrees of uncertainty or subjective probability here.

69 *The Equation* and the *defective truth table* are fundamental to the seman-  
70 tics of what has been called the conditional event ([21]) and the probability  
71 conditional ([1]), and we will use these terms equivalently in this paper<sup>2</sup>.  
72 There are philosophical and logical reasons ([31]), and empirical grounds  
73 ([36, 54, 55, 60]), for concluding that the indicative conditional of natural  
74 language is a probability conditional. [61] give theoretical reasons (going  
75 back to de Finetti [21, 22] and Ramsey [63]) and experimental support for  
76 closely comparing the natural language indicative conditional as a proba-  
77 bility conditional with a conditional bet in natural language. A bet on the  
78 natural language conditional, of the form *We bet that if A then C*, has three  
79 values. It is won when  $A \ \& \ C$  holds, lost when  $A \ \& \ \text{not-}C$  holds, and void  
80 or called off in *not-A* cases, and the probability of winning it, and the fair  
81 betting odds for it, is given by  $P(C \mid A)$ . In [61], approximately 80% of  
82 participants answer that the bet is called off when the antecedent is false  
83 (see also [26], p. 166, note 9 for similar results).

84  
85 From this point of view, to assert an indicative conditional is to perform  
86 a conditional speech act, a conditional assertion, which is like other condi-  
87 tional speech acts, a conditional bet or, for another example, a conditional  
88 promise, *We promise that if A then C*. These speech acts are void when  
89 *not-A* holds, in the sense that there is then no assertion, no bet, and no  
90 promise.

91  
92 There is, however, another way to look at the third value in a three-valued  
93 table that is well represented in logical and philosophical research on the  
94 conditional event and probability conditional, but not so far in psychology.  
95 In this view, the third value is seen as *uncertainty* (noted from now on '*U*').  
96 A state of uncertainty can of course easily arise for the categorical compo-  
97 nents, *A* and *C*, of an indicative conditional and a conditional bet. We can  
98 be uncertain whether *A* or whether *C* holds, and the result is that the truth  
99 table for both conditionals expands from a  $2 \times 2$  table to a  $3 \times 3$  table. Con-  
100 sider the example (similar to one in the [59] experiments):

101  
102 If the *USA* economy grows this year (*USA*), then the French economy will  
103 also grow (*FRA*).  
104

---

<sup>2</sup>However, [51] points out that logical validity can be defined in terms of probability, as suggested by *the Equation* ([31, 1]), or directly in terms of preserving values in de Finetti or other tables, and that these two types of definition validate different patterns of conditional inference.

105 We might have enough economic data on both countries to know that *USA*  
 106 and *FRA* are true or false, but we might also be waiting for data on either  
 107 country or both, keeping us uncertain about *USA* or *FRA* or both. Taking  
 108 the third value as uncertainty in this way, we can also have extended tables  
 109 for negation, *not-USA*, conjunction, *USA & FRA*, disjunction, *USA or FRA*,  
 110 and the material conditional, *not-(USA & not-FRA)*. One object of our ex-  
 111 perimental research has been to run the first psychological experiments in  
 112 which there is uncertainty about the components of natural language in-  
 113 dicative conditionals and bets on these, and on negations, conjunctions,  
 114 disjunctions, and the material conditional. These experiments can give us  
 115 evidence on how people classify these statement forms when they are uncer-  
 116 tain and give further support to the project of explaining reasoning under  
 117 uncertainty.

## 118 1. The defective table is not defective in the new paradigm

119 Wason [71] was the first psychologist to give truth table tasks about indica-  
 120 tive conditionals to ordinary people and to discover that their judgements  
 121 had three values (true *T*, false *F*, and irrelevant *I*).

		C	
<i>if A then C</i>		<i>T</i>	<i>F</i>
<i>A</i>	<i>T</i>	<i>T</i>	<i>F</i>
	<i>F</i>	<i>I</i>	<i>I</i>

Table 1. Participants' truth table built for *if A then C* (with *I* for irrelevant)

122 In [71], Wason did not use the term *defective* for the resulting Table 1. [46]  
 123 were apparently the first psychologists to use *defective* for the table 1, and  
 124 after their article, the rather negative term *defective* came to be used more  
 125 and more. Until very recently, the implication of most psychological research  
 126 on the *defective* table was that people were somehow *defective* in failing to  
 127 conform to a binary classification. There was no awareness shown in the  
 128 psychological literature, until the recent development of the new paradigm  
 129 ([3, 61]), that an identical table had been proposed by several philosophers  
 130 in their analysis of *if* in ordinary language with different interpretations of  
 131 the third value. Quine [62] (referring to Rhinelander) gives a table with a  
 132 value like irrelevance for a conditional with a false antecedent. O'Connor  
 133 [57] defines a table analogous to Table 1 with a third *undetermined* value.  
 134 Dummet [29] presents a table similar to Table 1 where the third value cor-  
 135 responds to *neither true nor false* (see also [45]). Kneale and Kneale [47],

136 suggest Table 1 where the value  $I$  characterizes a *gap* value<sup>3</sup>.

137

138 Following this point of view,  $I$  is considered as a third value that reflects a  
 139 state of *uncertainty*,  $U$ . We argue that participants' answers in truth-table  
 140 tasks yield a *coherent* table for the conditional under uncertainty, and there  
 141 is no supposedly *defective* table signifying a failure to understand the condi-  
 142 tional. This position is central to the new paradigm, but should be followed  
 143 by a full formal model of this coherent table with uncertainty. We must  
 144 specify the complete conditional truth table(s) and more generally define  
 145 the associated three-valued logic system(s).

146

147 We designed novel experimental materials to allow us to establish partic-  
 148 ipants' truth-tables for a conditional in which the antecedent and the con-  
 149 sequent could be true, false, or uncertain. As noted by Jeffrey [45], a table  
 150 similar to Table 1 does not fully characterise a three-valued truth-table for  
 151 the conditional *if A then C*, noted from now, following de Finetti's conven-  
 152 tion ([21]), as  $C \mid A$ . The  $U$  value in the body of Table 1, in the cases where  
 153 the antecedent is false, refers to a third value that must also be present in  
 154 the margins, as the antecedent or consequent can also be uncertain. Hence  
 155 Table 1 should be extended to a table with the following Table 2 format.

		C		
$C \mid A$		$T$	$U$	$F$
$A$	$T$	$T$	$?$	$F$
	$U$	$?$	$?$	$?$
	$F$	$U$	$?$	$U$

Table 2. The coherent truth-value table format for the conditional (with  $U$  for uncertain)

156 We presented participants in our experiment with a logically compound sen-  
 157 tence about a random chip, such as the natural language conditional *if the*  
 158 *chip is square then it is black*. The chips referred to could be in two colours,  
 159 black or white, and two shapes, square or round. In one scenario, the task  
 160 was to judge whether *if S then B* was true, false, or neither. In the other  
 161 scenario, the task was to judge whether bet on *if S then B* was won, lost,  
 162 or neither. In both scenarios, there were two conditions of visibility (rep-  
 163 resented on a computer). One, the chip was seen through a transparent

<sup>3</sup>In this gap interpretation, [47, p. 128] use also the term *defective* in a different sense than psychologists do: *defective* for *defective truth function*. In a similar gap interpretation Holdercroft uses the term *defective table* for Table 1 (see [44, p. 124]).

164 window, making  $S$  clearly true when the chip was square or clearly false  
 165 when  $S$  was round, and similarly for  $B$  and black or white. Two, the chip  
 166 was seen through a filtering window making it visually uncertain whether  
 167 the chip was square or round, or whether it was black or white. This tech-  
 168 nique allowed us to fill up the nine cells of a three-valued truth-table with  
 169 the participants' responses. The same materials were used for logical com-  
 170 pounds of the other connectives: negation  $not-S$ , conjunction  $S \mathcal{E} B$ , and  
 171 disjunction  $S \text{ or } B$ . And there was finally a material conditional expressed  
 172 in the form  $not-(S \mathcal{E} not-B)$ .

173

174 Another aspect of our work consisted of a comprehensive review of the  
 175 logical, linguistic, philosophical, and AI literatures on three-valued logics.  
 176 Probability theorists, particularly de Finetti [22], and other logicians and  
 177 philosophers have given normative reasons for adopting three-valued sys-  
 178 tems that include a conditional table with the Table 2 format ([50]). After  
 179 this review, we could compare our experimental results with the reviewed  
 180 systems. Before we describe our results, we will summarize our review of  
 181 the relevant three-valued tables in the normative literature.

## 182 2. Possible three-valued tables for the psychology of the in- 183 dicative conditional

184 Consider the range of possibilities for completing Table 2. The basic question  
 185 is what should be in the place of each '?' in Table 2:  $T$ ,  $U$ , or  $F$ ? There  
 186 are, in theory, 243 possible tables ( $3^5$ ). The same question arises for other  
 187 connectives - negation, conjunction disjunction, and the material conditional  
 188 - that can also be given general three-valued tables. Fortunately, we do  
 189 not start from a tabula rasa. Among numerous possible three-valued logics  
 190 ([19, 41, 67]), the formal literature contains nine three-valued logic systems  
 191 that are an extension of Table 1 for the conditional but also extend the  
 192 bi-valued logic for all other connectives. This is the connective that de  
 193 Finetti [21] called the conditional event and symbolized as  $C \mid A$ .<sup>4</sup> Such a  
 194 conditional has the fundamental property (I):

$$C \mid A = C \wedge A \mid A \tag{I}$$

---

<sup>4</sup>Any event  $C$  can be written in a conditional form  $C \mid T$ , where  $T$  is a tautology. Thus any three-valued connective table presented herein can be seen as a connective compounded with a conditional.

Three different extended conditional tables, which we call *de Finetti*, *Farrell* and *Cooper* conditional tables<sup>5</sup>, can be distinguished. These three tables can be used to categorise the nine systems.

## 2.1. Nine three-valued systems

All nine systems have involutive negation ( $\neg$ ). The conjunctive and disjunctive connectives are of four types: (i) Kleene-Lukasiewicz-Heiting (noted  $\wedge_K$  and  $\vee_K$ ), (ii) Sobociński (noted  $\wedge_S$  and  $\vee_S$ ), (iii) Bochvar (internal) (noted  $\wedge_B$  and  $\vee_B$ ) and (iv) McCarthy connectives (noted  $\wedge_M$  and  $\vee_M$ ). The systems explicitly incorporate a material conditional connective and thus also a material bi-conditional<sup>6</sup>. Six kinds of material conditional are proposed: (i) Kleene ( $\supset_K$ ), (ii) Lukasiewicz, ( $\supset_L$ ), (iii) Sobociński ( $\supset_S$ ), (iv and v) Bochvar (internal and external) ( $\supset_B$ ) and ( $\supset_{Be}$ ) and (vi) McCarthy ( $\supset_M$ ). A further selection of three-valued logic systems can be made taking into account the main properties we can reasonably expect the connectives to have.

### 2.1.1. The extended *de Finetti* conditional event table

*De Finetti*'s conditional table (see [21]) has been proposed and discussed by several authors. However, they have not always considered a comprehensive three-valued logic system with basic connectives (see for example [32, 52]). Strangely enough these authors failed to attribute the table to de Finetti, and they actually rediscovered the table with different interpretations of the  $U$  value, depending on their research field. We consider seven three-valued

---

<sup>5</sup>We call the tables using the name of the author who first proposed them. The *Farrell* and *Cooper* conditional tables are found in the IA literature and are often called *Goodman* and *Calabrese* conditional tables. Jeffrey in [45] proposes 16 possible conditional tables that respect some of Jeffrey's chosen properties. Among them, there are the *Farrell* and *Cooper* tables. However, if Jeffrey has an intuitionist negation, he does not specify the conjunctive, disjunctive and implication connectives. Consequently we have not considered Jeffrey's tables in our review.

<sup>6</sup>The fact that a system includes both the conditional event and the material conditional in a three-valued system is consistent with psychological results. Recent experiments of natural language conditionals support the new paradigm in showing that participants respond with a conditional event table. However, there is a minority of participants whose responses are consistent with the material conditional table. This raises the possibility that people can have two conditional interpretations: (i) The natural *default* interpretation would be the conditional event and (ii) a more specific/elaborated interpretation could be triggered by a particular context, e.g. definitional, logico-mathematic, and would consist in a material conditional interpretation.



		C			
		$C \mid_F A$	$T$	$U$	$F$
A	$T$		$T$	$U$	$F$
	$U$		$U$	$U$	$U$
	$F$		$U$	$U$	$U$

Table 3. The extended *de Finetti* conditional event table  $C \mid_F A$ 

logic systems in this category<sup>7</sup>.

**Fi system.** This system includes the conjunction  $\wedge_K$  and disjunction  $\vee_K$  as well as material conditional ( $\supset_K$ )<sup>8</sup>. It has been expounded in at least five different ways<sup>9</sup>.

- i De Finetti defined **Fi** as a logic of probability that is a *superimposed* logic on a two-valued system (see [21, 23, 24, 25]). The third value represents doubt or uncertainty in an individual who is wondering whether a proposition is true or false. An event is always true or false, but there is a third case when the individual lacks the relevant knowledge and is uncertain. It is a ‘transitory’ subjective state of the individual, and the three-valued classification can become two-valued as knowledge increases. In this way,  $U$  is interpreted as a kind of ‘transitory’ value and not a third non-subjective value of the same type as truth or falsity<sup>10</sup>;
- ii Hailperin (see [42, 43]) introduces **Fi** as the logic that supports conditional probability logic.  $C \mid_F A$  is illustrated with a suppositional interpretation: *If A then C* interpreted as  $C$ , *supposing A* or *supposing A, then C*. The value  $U$  represents the *undetermined, unknown*

<sup>7</sup>We regroup in the ‘same system’ the systems that propose the same set of connectives. The truth tables for the connectives are displayed in the Appendix A (see Tables 7 to 10).

<sup>8</sup>The bi-conditional connective  $\Leftrightarrow_K$  is not often explicit

<sup>9</sup>Recently [68] endorses the same **Fi** system quoting de Finetti.

<sup>10</sup>The conditional event table is illustrated by a conditional bet interpretation: *If A then C* interpreted as a *bet that if A then C*. Thus if  $A$  and  $C$  are true the bet is won, if  $A$  is true and  $C$  is false the bet is lost and if  $A$  is false or  $A$  or  $C$  are unknown the bet is called off. Such a void or null bet could be seen as having a kind of extreme uncertainty. In de Finetti’s system, there are two additional connectives that allow to return to the bi-valued logic: a *Thesis* connective  $T(A)$  which means ‘ $A$  is true’, and a *Hypothesis* connective  $H(A)$  which means ‘ $A$  is not null’,  $X = C \mid_F A$ ;  $T(X) = C \wedge A$  and  $X = T(X) \mid_F H(X)$  (see in the Appendix A, the Table 6).

- 235 or of no interest value;<sup>11</sup>
- 236 iii Blamey (see [12]) proposes **Fi**, as a *simple partial logic*, where the  
 237 third value  $U$  is interpreted as a truth-value gap that is considered  
 238 as the minimal value (whereas the two truth-values true and false  
 239 cannot be compared to each other). In this system  $C \mid_F A$  is called  
 240 the *transplication*<sup>12</sup>;
- 241 iv In the linguistic field, Beaver formalize an identical **Fi** system ([7,  
 242 8, 9]).  $C \mid_F A$  is used as an ‘elementary presupposition operator’,  
 243 defined with a ‘unary operator’  $\partial$ <sup>13</sup>.
- 244 v Rescher (see [66, 67]) discusses a *quasi-truth functional* system that  
 245 is exactly **Fi** but where  $U$  is defined as an *undetermined* value (the  
 246 bracketed entry  $(T, F)$  that can be either  $T$  or  $F$  depending on the  
 247 circumstances).
- 248 **R system.** The second system is called **R** for Reichenbach’s *quantum system*  
 249 ([64, 65]). It includes as **Fi** the conjunction  $\wedge_K$  and disjunction  $\vee_K$  but  
 250 uses two material conditionals:  $\supset_L$  and  $\supset_{Be}$ <sup>14</sup>.  $C \mid_F A$  is called *quasi-*  
 251 *implication* and can be represented as the observation of an experiment  
 252 that is true if observation  $A$  has given the result  $C$ , false if observation  
 253  $A$  has given the result *not* –  $C$  and is meaningless or indeterminate if  
 254 the observation  $A$  has not been made. It is very close to de Finetti’s  
 255 conditional bet interpretation (see for a discussion the Appendix of [23]).
- 256 **BG system.** This third system developed by the mathematicians Bruno and  
 257 Gilio ([13]), shares de Finetti’s bet interpretation of the conditional. The  
 258 disjunction table corresponds to  $\vee_S$  and the conjunction to  $\wedge_B$ .
- 259 **BF system.** The fourth system has been introduced in the field of logic (the  
 260 logic of assertion for Belnap [10], and of presupposition for Farrell [37]).  
 261 In **BF** system, the conjunction and the disjunction correspond to  $\wedge_S$  and  
 262  $\vee_S$ .

<sup>11</sup>Hailperin introduces a connective  $\Delta$  which means *don’t care* and can be used to define  $C \mid_F A$  thus  $C \mid_F A = \Delta(\neg A \vee (A \wedge C)) = \max\{\min A, C\}, \min\{1 - A, U\}$  with  $F < U < T$ .

<sup>12</sup>In his system, Blamey uses in addition an *interjunction* connective (noted  $\times$ ) defined by  $A \times C = (A \wedge_K U) \vee_K (A \wedge_K C) \vee_K (U \wedge_K C) = (A \vee_K U) \wedge_K (A \vee_K C) \wedge_K (U \vee_K C)$ . The conditional can be defined in relation with the *interjunction*  $C \mid_F A = [A \wedge_K C] \times [A \supset_K C]$  (see Table 6).

<sup>13</sup> $C \mid_F A = (\partial(A) \wedge_K C) \vee_K (\partial(A) \vee_K \partial(\neg A))$  (see Table 6).

<sup>14</sup>Reichenbach calls  $\supset_{Be}$  *alternative* material condition. He adds to the involutive negation (called by Reichenbach *diametrical negation*), two other negations (*Cyclical*  $\sim A$  and *Complete*  $\bar{A}$ ).

263 **Mc system** The Mc system supported by McDermot ([49]) groups Fi and  
 264 BF systems (there are two conjunctions  $\wedge_K$  and  $\wedge_S$  and two disjunctions  
 265  $\vee_K$  and  $\vee_S$  together with de Finetti's conditional bet interpretation.  
 266 **MBV system.** The seventh system proposed by Muskens, Van Benthem and  
 267 Visset ([53]) in the linguistics field takes the conjunction  $\wedge_M$  and the  
 268 disjunction  $\vee_M$ .

### 269 2.1.2. The extended *Farrell* conditional event table

270 In the literature, there is one three-valued logic system that includes the  
 271 *Farrell* conditional table.

		C			
		$C \mid_{Fa} A$	$T$	$U$	$F$
A	$T$	$T$	$T$	$U$	$F$
	$U$	$U$	$U$	$U$	$F$
	$F$	$U$	$U$	$U$	$U$

Table 4. The extended *Farrell* conditional event table  $C \mid_{Fa} A$

272 **GNW system.** In a logical approach, with  $U$  standing for *inappropriate*, Far-  
 273 rell in [38] proposes  $C \mid_{Fa} A$  and  $\wedge_K$  and  $\vee_K$  for conjunction and disjunc-  
 274 tion<sup>15</sup>. This system, called from now on **GNW**<sup>16</sup>, has been independently  
 275 proposed by Goodman and colleagues in an algebraic approach where  
 276  $C \mid_{Fa} A$  corresponds to a coset that can also be represented by an interval  
 277  $[A \wedge C, \neg C \vee A]$  (see for example [40]).

### 278 2.1.3. The extended *Cooper* conditional event table

279 Two three-valued logic systems include the *Cooper* conditional table.

280 **SAC system.** SAC system<sup>17</sup> was initially proposed independently by Cooper  
 281 ([20]) and Belnap ([11]) in the field of logico-linguistics and logic, respec-

<sup>15</sup>Farrell (1979) introduces also a bi-conditional-equivalence operator based on the conditional table (see also section 2.2.3).

<sup>16</sup>In the AI literature, this system is called **GNW** for Goodman, Nguyen and Walker (see for example [18]). For parsimony we adopted also **GNW**.

<sup>17</sup>In the AI literature, this system is called **SAC** for Schay, Adams and Calabrese. Adams and Schay are associated to this system because both authors independently introduced the connectors *quasi conjunction* and *quasi disjunction* that are actually the Sobociński connectives. However these authors have never (to our knowledge) given explicitly the *Cooper* conditional table (with the 9 cells) or an iterated rule of conditional that allows to

		C			
		$C \mid_C A$	$T$	$U$	$F$
A	$T$	$T$	$T$	$U$	$F$
	$U$	$T$	$U$	$U$	$F$
	$F$	$U$	$U$	$U$	$U$

Table 5. The extended *Cooper* conditional event table  $C \mid_C A$ 

282 tively (see also [30]). The conditional is defined as conditional assertion,  
 283 and the third value  $U$  means unassertive.  $\wedge_S$  and  $\vee_S$  define the system.  
 284 Independently, Calabrese in numerous papers (see for example [14, 15]  
 285 adopts an algebraic approach to the conditional and defines an identical  
 286 system where the third truth-value  $U$  is used for an *inapplicable* condi-  
 287 tional.

288 **Ca system.** Cantwell defines a second system, **Ca** from now on, that sup-  
 289 ports  $C \mid_C A$  with  $\wedge_K$  and  $\vee_K$  as conjunction and disjunction (see  
 290 [17, 16]. The third value is interpreted here as a *gap* value.

## 291 2.2. Three valued-logic systems and three basic constraints

### 292 2.2.1. The Bayesian *conditioning* constraint on the conjunctive 293 connective

294 Some intuitive constraints on the conjunctive connective  $\mathcal{E}$  have been for-  
 295 mulated by [28]: The conjunction connective must respect the five following  
 296 constraints: (i) it extends the bi-valued conjunction of propositional logic  $\wedge$ ,  
 297 (ii) the conjunction of two uncertain conditionals must be uncertain, (iii) it  
 298 is commutative and (iv) the conjunction between uncertain and true must  
 299 be either true or uncertain and (v) it must follow Bayesian conditioning:

$$P(C \wedge A) = P(C \mid A)P(A) \quad (\text{II})$$

300 Calling  $t(A)$  a usual truth-assignment function, [28] show that the logical  
 301 version of (II) is

$$t(C \mid A \mathcal{E} A \mid T) = t(C \wedge A \mid T) \quad (\text{III})$$

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construct the *Cooper* conditional table. Note also that Schay ([69]) proposes two additional connectives for conjunction and disjunction that are the Bolchvar (internal) connectives.

Now if  $A$  is false,  $C \wedge A$  is false then  $C \mid A$  is equal to  $U$  (for the three connectives tables):

*The conjunction of false and uncertainty must be false.*

Whereas the four types of conjunctive connectives related to our nine three-valued logic systems verify the first four constraints, two of them fail to meet the fifth constraint: The Bochvar (internal) conjunctive  $C \wedge_B$  and McCarthy conjunctive  $C \wedge_M$  do not respect this constraint. For these two connectives we have:  $F \wedge_B U = F \wedge_M U = U$ . The two three-valued logic systems **BG** and **MBV** that include Boschvar's (internal) and McCarthy's conjunctions can be removed from our scope.

### 2.2.2. Basic constraint on the *order* of the truth-values

A traditional interpretation of disjunctive and conjunctive connectives in the bi-valued logic is to assume an order on the two truth-values  $F < T$ . Thus the conjunction connective corresponds to a minimum and the disjunction connective to a maximum. The Łukasiewicz-Kleene (strong)-Heiting and Sobociński conjunction and disjunction connectives can also be formulated as a minimum and a maximum. In this way the truth-value order for Łukasiewicz-Kleene (strong)-Heiting connectives is  $F < U < T$ .

In Sobociński's conjunctive connective, the order is  $F < T < U$ . However if we interpret the Sobociński disjunction as a maximum, the order must be modified:  $U < F < T$ . This absence of symmetry can be theoretically allowed (see [28]) but seems difficult to justify from a psychological point of view.

### 2.2.3. Basic constraints on the *equivalence* connective

As mentioned above, all the nine three-valued systems initially distinguished include a conditional event and a material conditional connective. Except for Farrell ([38]), no author has proposed a bi-conditional event based on the conditional event. However, from a psychological point of view, if people naturally interpret the natural language conditional following the conditional event table, they must also interpret the natural bi-conditional as the conjunction of two conditional events. Thus it seems also important to include this additional connective called from now on the *equivalence* connective (noted  $||$ ). From a Bayesian point of view, the probable equivalence between two events  $A$  and  $C$  is formulated by the following relation ([48]):

$$P(C \parallel A) = \frac{P(C \wedge A)}{P(C \vee A)} = P(C \wedge A \mid C \vee A) \quad (\text{IV})$$

337 which is supported by experimental data (see for example [48, 70]). By  
 338 analogy, we can expect that the equivalence connective verifies a similar  
 339 relation:

$$C \parallel A = C \mid A \wedge A \mid C = C \wedge A \mid C \vee A \quad (\text{V})$$

340 Lets consider if this equality is respected for each equivalence connective in  
 341 the seven left three-valued logic systems (see Appendix A)).

- 342 • In the **Fi** and **R** systems the equivalence connective  $C \parallel_F$  for  $(C \mid_F$   
 343  $A) \wedge_K (A \mid_F C)$  based on the de Finetti  $\mid_F$  conditional corresponds to  
 344  $(C \wedge_K A) \mid_F (C \vee_K A)$  (see Table 11)<sup>18</sup>.
- 345 • The equivalence connective  $C \parallel_{Fa} A$  built on the *Farrell* conditional  
 346  $C \mid_{Fa} A ((C \mid_{Fa} A) \wedge_K (A \mid_{Fa} C))$  for the **GNW** system is equal to  $(C \wedge_K$   
 347  $A) \mid_{Fa} (C \vee_K A)$  (see Table 11).
- 348 • Among the two systems that include the *Cooper* conditional, only the  
 349 equivalence of **Ca**  $C \parallel_{Ca} A$  is equal to  $(C \wedge_K A) \mid_C (C \vee_K A)$  (see Tables  
 350 11 and 14)<sup>19</sup>.

351 To summarize, only three three-valued logic systems respect the three con-  
 352 straints on conjunction, order, and equivalence. They are the **Fi**, **R**, and  
 353 **GNW**. The first two systems support *de Finetti* conditional event table and  
 354 their difference lies only in the material conditional. Both systems have a  
 355 very similar interpretation of the conditional event: the consequent  $C$  is  
 356 the result of a dynamic process: a bet for de Finetti and an experiment  
 357 for Reichenbach. **GNW** proposes the *Farrell* conditional table but shares all  
 358 other connectives with de Finetti's three-valued logic system. In **GNW**, the  
 359 conditional is either an conditional assertion ([38]) or a mathematical object  
 360 similar to an interval ([40]).

<sup>18</sup>We can note that  $C \parallel_F$  is also equal to the  $(C \wedge_B A) \mid_F (C \vee_S A)$  that is not equal to  $C \parallel_{BG} A$  (see Table 12). It is the same result with **MBV** system;  $(C \wedge_M A) \mid_F (C \vee_M A)$  is equal to  $C \parallel_F$  but not to  $C \parallel_{MBV}$  (see Table 12). We can also note that  $C \parallel_{FB}$  is equal to  $C \parallel_F$  but not to  $(C \wedge_S A) \mid_F (C \vee_S A)$  (see Table 13). Thus for **Mc** we only have equality when it is reduced to a **Fi** system.

<sup>19</sup>It is also identical to  $C \parallel_{Fa} A$  (see Table 11).

### 3. The three-valued systems and the experimental results

As noted in section 1, the participants in our experiment made judgements in two scenarios about logically compound statements: negation *not-A*, conjunction *A & C*, disjunction *A or C*, the natural language conditional *if A then C*, and the material conditional in the form *not-(A & not-C)*. There was an assertion scenario, where participants were asked to judge whether a statement was true or false. And there was a bet scenario, where participants were asked to assess whether a bet was won or lost. A novel aspect of our experiment was that the component statements, *A* and *C*, could be uncertain as well as the compound statements. The statements referred to chips that had a round or square shape and a black or white colour. The type of uncertainty we studied was visual: a filter could block the sight of the shape or colour of a chip.

Our main results can be briefly summarized ([4]). Our first main result was that the participants' responses were parallel in the two scenarios - the assertion and the bet - for all connectives we reviewed. People treat questions about the truth or falsity of assertions as similar to questions about winning or losing bets, and in particular, they treat natural language conditional assertions as similar to conditional bets. This result confirms, at a much more general level, the findings of [61]). The second main finding was that people agreed on their interpretation of negation, conjunction, and disjunction (see below), but were not unanimous on the natural language conditional, *if S then B*. For *if S then B*, the two main answers correspond to the conditional table 2 of section 1 (see Table 2 ) and to the conjunction table. This finding confirms, again at a more general level, previous research showing that some people have a conjunctive interpretation of the conditional for the type of materials we used here. There is evidence that this interpretation is the result of processing limitations (see [3] for a discussion of this evidence).

We have analysed the complete set of tables given by the participants' responses and have categorized these by how close they were to the tables we reviewed from the normative literature ([67]), which we summarized above in Section 2.1.<sup>20</sup> The first significant outcome of this analysis is that most

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<sup>20</sup>The proximity corresponds to the number(s) of cell difference(s) with coherent tables of the literature. For example if the conditional table has 0 difference with de Finetti's conditional table, it is classified 'de Finetti'. If it has only one difference with de Finetti's conditional table and that this is the smaller 'distance' it is also classified in the de Finetti

of the participants' tables can be classified using the three-valued normative tables of Section 2.1. Their responses were not scattered over the 243 possibilities. Participants treated uncertainty coherently. In more detail, nearly all participants reproduced the involutive negation table. For the other connectives, the majority of responses coincided with the three-valued tables of the **Fi** logic system. These results for conjunction and disjunction confirm our expectations stated in section 2.2 on the connectives  $\wedge_K$  and  $\vee_K$ . Recall that only the three systems (**Fi**, **R** and **GNW**) respect basic constraints. A second significant fact was the modal responses respected de Finetti's conditional event table, not only for the conditional bet condition but also for the conditional assertion. Most participants did not treat the natural language conditional and the material conditional as similar to each other. These results add to the mounting evidence that ordinary people interpret the natural language indicative conditional as very close to de Finetti's conditional event.

## Conclusion

For several decades psychologists have known that people judge that *if A then B* is true when *A* holds and *B* holds, false when *A* holds and *B* does not, and neither true nor false when *A* does not hold. For the last decade, there has been growing psychological evidence that people judge that the probability of the indicative conditional,  $P(\text{if } A \text{ then } C)$ , is the conditional probability of *C given A*,  $P(C | A)$ . More recently, psychologists have shown that there is a close relation between indicative conditionals and conditional bets. There is a great need to integrate these experimental findings in the new paradigm in the psychology of reasoning, with its Bayesian point of view. In our view, integration has been held back because psychologists did not even raise the general question of which three-valued tables people's judgements correspond to under uncertainty. We have raised this general question and have systematically reviewed the relevant three-valued systems from the normative literature. We have also indicated how we investigated experimentally which normative tables provide the best descriptive fit for people's judgements under uncertainty. We are not of course trying to make normative or logical judgements about which three-valued system should be preferred for some given interpretation of the third value. Our aim is to advance the new paradigm in the psychology of reasoning and its goal of a Bayesian account of ordinary reasoning. The result of our investigation is

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bucket.



432 support for de Finetti's three-valued tables in general, and his conditional  
433 event table in particular, as descriptive of people's judgements under uncer-  
434 tainty.

435 **A. Appendix: Three-valued truth tables**

$A$	$\neg A$	$\sim A$	$\overline{A}$	$T(A)$	$H(A)$	$\Delta(A)$	$\partial(A)$
T	F	U	U	T	T	U	T
U	U	F	T	F	F	U	U
F	T	T	T	F	T	F	U

Table 6. The truth tables for the involutive negation,  $(\neg A)$ , Reichenbach ‘cyclic negation’  $(\sim A)$ , Reichenbach’s ‘complete’ negation  $(\overline{A})$ , the unary de Finetti’s Thesis connective  $(T(A))$ , de Finetti’s hypothesis connective  $(H(A))$ , Hailperin’s unary connective ‘don’t care’  $(\Delta(A))$ , Blamey’s unary ‘presupposition operator’  $(\partial(A))$ .

$A$	$C$	$A \vee_K C$	$A \wedge_K C$	$A \supset_K C$	$A \Leftrightarrow_K C$	$A \supset_L C$	$A \Leftrightarrow_L C$
T	T	T	T	T	T	T	T
T	U	T	U	U	U	U	U
T	F	T	F	F	F	F	F
U	T	T	U	T	U	T	U
U	U	U	U	U	U	T	T
U	F	U	F	U	U	U	U
F	T	T	F	T	F	T	F
F	U	U	F	T	U	T	U
F	F	F	F	T	T	T	T

Table 7. Łukasiewicz–Heyting–Kleene’s truth tables for disjunction  $(\vee_K)$  and conjunction  $(\wedge_K)$ , Keene’s truth table for implication  $(\supset_K)$ , Kleene–Bochvar–McCarthy’s truth table for bi-conditional  $(\Leftrightarrow_K)$ , Łukasiewicz’s truth tables for implication  $(\supset_L)$  and bi-conditional  $(\Leftrightarrow_L)$ .

$A$	$C$	$A \vee_B C$	$A \wedge_B C$	$A \supset_B C$	$A \supset_{Be} C$	$A \Leftrightarrow_R C$
T	T	T	T	T	T	T
T	U	U	U	U	F	F
T	F	T	F	F	F	F
U	T	U	U	U	T	F
U	U	U	U	U	T	T
U	F	U	U	U	T	F
F	T	T	F	T	T	F
F	U	U	U	U	T	F
F	F	F	F	T	T	T

Table 8. Bochvar's truth tables for disjunction ( $\vee_B$ ), conjunction ( $\wedge_B$ ), implication ( $\supset_B$ ), Bochvar-Reichenbach's truth tables for 'alternative' implication ( $\supset_{Be}$ ) and bi-implication ( $\Leftrightarrow_R$ ).

$A$	$C$	$A \vee_S C$	$A \wedge_S C$	$A \supset_S C$	$A \Leftrightarrow_S C$
T	T	T	T	T	T
T	U	T	T	F	F
T	F	T	F	F	F
U	T	T	T	T	F
U	U	U	U	U	U
U	F	F	F	F	F
F	T	T	F	T	F
F	U	F	F	T	F
F	F	F	F	T	T

Table 9. Sobociński's truth tables for disjunction ( $\vee_S$ ), conjunction ( $\wedge_S$ ), implication ( $\supset_S$ ) and bi-conditional ( $\Leftrightarrow_S$ ).

$A$	$C$	$A \vee_M C$	$A \wedge_M C$	$A \supset_M C$
T	T	T	T	T
T	U	T	U	U
T	F	T	F	F
U	T	U	U	U
U	U	U	U	U
U	F	U	U	U
F	T	T	F	T
F	U	U	F	T
F	F	F	F	T

Table 10. McCarthy's truth tables for disjunction ( $\vee_M$ ), conjunction ( $\wedge_M$ ) and implication ( $\supset_M$ ).

$A$	$C$	$A \parallel_F C$ $= (A \mid_F C) \wedge_K (C \mid_F A)$ $= (C \wedge_K A) \mid_F (C \vee_K A)$	$A \parallel_{Fa} C$ $= (A \mid_{Fa} C) \wedge_K (C \mid_{Fa} A)$ $= (C \wedge_K A) \mid_{Fa} (C \vee_K A)$ $= A \parallel_{Ca} C$ $= (A \mid_C C) \wedge_K (C \mid_C A)$ $= (C \wedge_K A) \mid_C (C \vee_K A)$
T	T	T	T
T	U	U	U
T	F	F	F
U	T	U	U
U	U	U	U
U	F	U	F
F	T	F	F
F	U	U	F
F	F	U	U

Table 11. Equivalence truth tables for **Fi** system and **R** system ( $\parallel_F$ ) for **GNW** system ( $\parallel_{Fa}$ ) and for **Ca** system ( $\parallel_{Ca}$ ).

$A$	$C$	$A \parallel_{BG} C$ $= (A \mid_F C) \wedge_B (C \mid_F A)$ $= A \parallel_{MBV} C$ $= (A \mid_F C) \wedge_M (C \mid_F A)$	$(C \wedge_B A) \mid_F (C \vee_S A)$ $= A \parallel_F C$ $= (C \wedge_M A) \mid_F (C \vee_M A)$
T	T	T	T
T	U	U	U
T	F	U	<u>F</u>
U	T	U	U
U	U	U	U
U	F	U	U
F	T	U	<u>F</u>
F	U	U	U
F	F	U	U

Table 12. Equivalence truth tables for **BG** system ( $\parallel_{BG}$ ) and **MBV** system ( $\parallel_{MBV}$ ).  $\parallel_{BG}$  is not equal to  $(C \wedge_M A) \mid_F (C \vee_M A)$  and  $\parallel_{MBV}$  is not equal to  $(C \wedge_B A) \mid_F (C \vee_S A)$ .

$A$	$C$	$A \parallel_{BF} C$ $= (A \mid_F C) \wedge_S (C \mid_F A)$ $= A \parallel_F C$	$(C \wedge_S A) \mid_F (C \vee_S A)$
T	T	T	T
T	U	U	<u>T</u>
T	F	F	F
U	T	U	<u>T</u>
U	U	U	U
U	F	U	U
F	T	F	F
F	U	U	U
F	F	U	U

Table 13. Equivalence truth tables for BF system ( $\parallel_{BF}$ ). It is not equal to  $(C \wedge_S A) \mid_F (C \vee_S A)$ .

$A$	$C$	$A \parallel_{SAC} C$ $= (A \mid_C C) \wedge_S (C \mid_C A)$	$(C \wedge_S A) \mid_C (C \vee_S A)$ $= (C \wedge_S A) \mid_F (C \vee_S A)$
T	T	T	T
T	U	T	T
T	F	F	F
U	T	T	T
U	U	U	U
U	F	F	<u>U</u>
F	T	F	<u>F</u>
F	U	F	<u>U</u>
F	F	U	U

Table 14. Equivalence for SAC system is not equal to  $(C \wedge_S A) \mid_C (C \vee_S A)$ .

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